Finding Transfer Functions

Padron, Sebastian & Reyes, David
Group #18 Date: 2/11/2024

***Abstract*—The primary objective of this lab report is to experimentally measure the performance of a motor flywheel system through different system responses and to derive a transfer function and state-space models which are representative of the systems. There will be two methods, the time-domain approach, which uses a step input, and frequency-domain approach, which uses sinusoidal waves as its input signal. Both approaches will generate values in order to determine the time constant, DC gain and pole location to generate the transfer function of the system. Two independent sets of data are compared and used to create Bode plots to display the similarities between the models. Both showed elements of first-order systems in the experimental plots. Despite both sets of data make use of different motor efforts, the documented DC gain and time constant values for the sinusoidal response are 6.1798-6.2140 dB and 0.07967-0.07958 s, respectively, and the documented DC gain and time constant values for the step response are 6.3904-6.7407 dB and 0.0424-0.0442 s, respectively.**

***Index Terms*— Bode Plot, DC Gain, Time Constant, Transfer Function**

I. Introduction

THE formulation of transfer functions or state-space models using real experimental data is crucial in quantitatively describing a control system's dynamics. In this report a motor flywheel device will be the primary focus in developing transfer functions in the Laplace domain. The procedure will make use of the considerable amount of experimental data which will provide information on the performance of the device and subsequently its physical parameters.

To determine these physical parameters, two methods are available, with the first being the time-domain approach which calls for the driving of the system with step inputs where the signal generator produces square waves and measures the response of the system. The other method is the frequency-domain approach which is driven by an input of multiple sinusoidal frequencies where a response is characterized. Here the direct current (DC) gain and the system phase shift which provides information on the system's response time will be plotted in a bode plot to relate the experimental results to the system model. A bode plot is a logarithmically scaled plot that will simplify the gathering of the proper terms needed for the transfer function.

The report will complete the different methods, gathering and comparing the two sets of independent data and their transfer functions. The system used in this experiment is a Furuta Pendulum system. It involves a DC gearmotor with a 3:1 belt drive and a ball-bearing supported platen, of which both have a single degree of freedom [1].

To conduct sinusoidal and square waves for the velocity output, a 12-volt DC power supply is installed to drive the physical rotation of the platen. The motor itself is instructed with input pulse width modulation command signals sent from a computer, that ranges from 0 to 1023 altering the effort from the motor. To record the data, a USB Data Acquisition Device (DAQ) is used and sent to LabView where it translates the electronic signals into comprehensible values, in this case the angular velocity of the motor flywheel system.

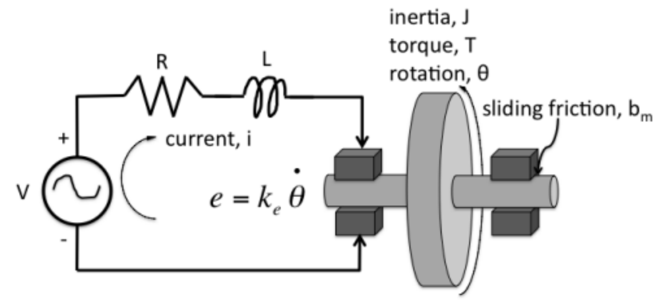

Fig. 1. The lumped parameter circuit model of a DC motor.

Within this report, the system motor is represented in a lumped parameter circuit model as shown in Fig. 1. Here the circuit considers the resistance (R) and inductance (L) terms, the rotational inertia (J), motor torque constant (τ) and the motor friction (b_m). Given the definition of a lumped parameter, all sources of friction and moving parts are characterized by a single term, the back-electromotive constant (k_e) and the torque constant (k_T) respectively [3]. With these motor values, a transfer function $(\mathrm{G}(s))$ in the Laplace domain is generated as a function of the angular velocity in radians per second $(\omega(s))$ over the input command signal $(V(s))$.

$$G(s) = \frac{\omega(s)}{V(s)} = \frac{k_T}{(Js + b_m)(Ls + R) + k_e k_T} \qquad (1)$$

II. Methods

A. Equipment Preparation

For this lab, the required equipment as mentioned includes a laptop installed with LABVIEW, a multi-function DAQ, a 12-volt power supply, and a constructed Furuta Pendulum system which includes a DC motor. This motor flywheel device followed the "Pendulum v2.1 Assembly" instructions which were provided in lab [2].

B. Software Preparation

Following the assembly of the motor flywheel system a signal generator is added to the standard lab software on LabView to provide sine and square waves input signals for the motor. The virtual instrument is then modified to fulfill experimental needs. A command signal is wired to the motor output, with a Butterworth lowpass filter on the platen rotation angle to give clearer data and better estimates. Then with a derivation function on the platen rotation angle, the rotational velocity of the system can be obtained to generate a voltage versus velocity Bode Plot. This rotational velocity along with the frequency, amplitude, signal type, command signal and elapsed time is saved into a spreadsheet that can later be exported onto a data manipulation software such as Microsoft Excel.

C. Data Collection

Prior to collecting the data from either approach, a dithering compensation (i.e., the Turn on Command) is implemented to counteract the static and dynamic friction. To determine this value, the signal generator is set to give a sinusoidal input with an amplitude of 500 and a frequency of 0.25 hertz. With this input signal the turn on command value is increased until the researcher claims a smooth sinusoidal velocity output without slope discontinuities in its data [1]. Through this procedure Researcher 1 and Researcher 2 resulted in a turn on command value of 90 and 69 respectively.

Following the determination of the dithering compensation, the amplitude is then set and maintained for continuity to being with the frequency-domain approach. Researcher 1 set their amplitude to a magnitude of 300 and Researcher 2 set their amplitude to a magnitude of 400. These values directly translate to the amount of effort provided by the motor. Then, with these constant set values, each researcher will accumulate data through various frequencies. These frequencies include 0.05, 0.1, 0.2, 0.5, 1, 2, 5, 7.5, 10, 15, and 20 hertz. It is critical to take a substantial amount of data in order to have accurate results. Consequently, a minimum of five cycles was taken for each individual frequency and logged continuously.

The time-domain approach was set-up in the same manner as the frequency-domain approach with the same turn on command value and amplitude, however the input signal generator is generating square waves and only taking one set of data at a frequency of 0.2 hertz.

D. Data Manipulation

Under the assumption that the recorded data was gathered and exported properly onto a data manipulation software, the data is then filtered further to remove excessive noise and the adjustment time between frequencies in order to provide consistent and accurate results. A series of unit conversions required to create a performing Bode plot will follow.

The raw input command signal is converted to voltage for every frequency as shown [4].

$$V(s) = (Command\ Signal) \times \frac{9\ V}{1023} \qquad (2)$$

The output platen velocity is converted from its original form of degrees per second to radians per second.

$$\omega(s) = (Platen\ Velocity) \times \frac{\pi}{180} \qquad (3)$$

It should also be noted that for the bode plot, the frequency was also converted to radians per second to then plot on the x-axis by multiplying the frequency by 2π.

E. Bode Plot Generation – Frequency Domain Approach

The bode plots that will be created for the frequency-domain approach will require the equations for the magnitude of the DC gain and phase shift for each frequency. The gain plot takes the ratio of the maximum output value to the maximum input value within roughly the same period to obtain its magnitudes for the different frequencies.

$$Gain = 20 \times \log_{10}\left(\frac{\omega_{max}(s)}{V_{max}(s)}\right) \qquad (4)$$

The phase plot will utilize the zero-crossing time (ZCT) of the input signal and the output velocity which is obtained through the observation of the values crossing zero on their corresponding y-axes. These time differences created a time delay between the input and output which is used to calculate the phase shift.

$$Phase = 360 \times (Frequency) \times \Delta t \qquad (5)$$

F. Step Response – Time Domain Approach

Due to the only set of data coming from the low frequency of 0.2 hertz, the determined gain will be the maximum value of the square wave. Given that the input is a step response this value will be roughly equal to the gain of the system. The phase shift is obtained by the amount of time it takes the value to rise 63.2% of the graph. Given that realistically there is time between the shifts in the velocity output, the data can easily display the time delay in the hardware keeping it from instantaneously switching directions.

G. Theoretical Models

With the values for gain and phase for every frequency in the frequency-domain approach appropriately set, bode plots are generated giving information on the DC gain (K_{DC}) and time constant (τ). The time constant in this case is the inverse value of the frequency where the graph deviates from the horizontal asymptote that is seen in beginning of the graph. It is the point where the graph begins to decline on a negative slope. It is with these values, a first-order transfer function is produced to create the simulated response in MATLAB to then find the theoretical transfer function ($G_{theoretical}(s)$) where it will then be compared to the experimental plot and its corresponding transfer function.

$$G_{theoretical}(s) = \frac{K_{DC}}{\tau s + 1} \qquad (6)$$

For the theoretical model of the time domain approach the gain is the aforementioned maximum output value, and the time constant is the inverse of the time it took to ascend 63.2% of the graph when the rotating platen switches direction [3]. Moreover, these values will once again be implemented into (6) to generate a theoretical transfer function to compare to the experimental results.

III. Results

A. Experimental Responses

Initially, the sinusoidal response of the system is evaluated at different input frequencies to produce a bode plot from experimental data. The bode plot is split into a magnitude plot, shown in Fig. 2, and a phase plot, shown in Fig. 3.

The magnitude plot depicts the amplification or attenuation of the response at different frequencies. Both data sets from Researcher 1 and Researcher 2 exhibit a change of slope from 0 to -10 decibels per decade at around a frequency of 12 radians per second.

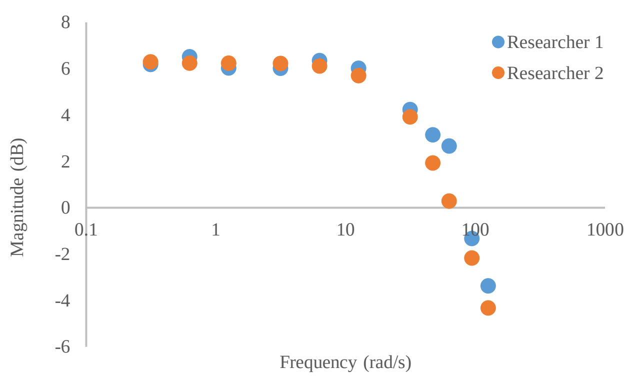

Fig. 2. Experimental magnitude plot of the sinusoidal response with a logarithmic frequency scale.

The phase plot in Fig. 3 shows the shift or time delay introduced by the output sinusoidal response at different frequencies. Both of the data sets from Researcher 1 and Researcher 2 present a horizontal asymptote around at a phase angle of 0 degrees until around a frequency of 12 radians per second. After the fluctuation point, the data set from Researcher 1 begins with a slope of -200 degrees per decade. Conversely, the data set from Researcher 2 begins with a slope of -400 degrees per decade after the fluctuation point i.e., the pole location.

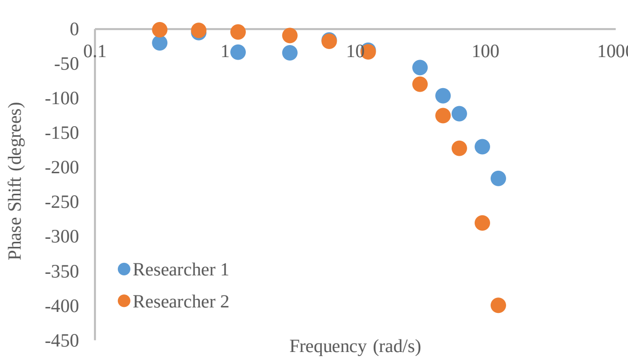

Fig. 3. Experimental phase plot of the sinusoidal response with a logarithmic frequency scale.

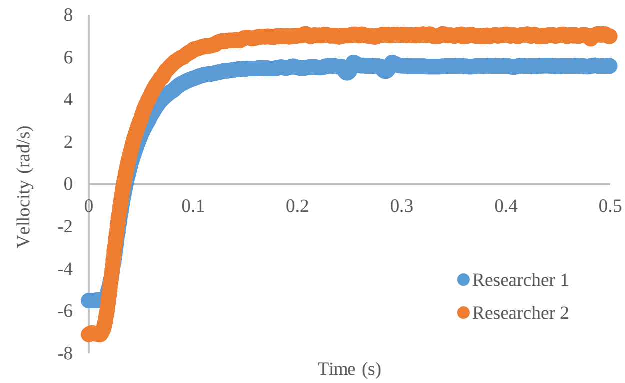

Fig. 4. Plot of the experimental step response in the time domain. Time delay due to hardware is displayed at the start of each data set.

TABLE I
Researcher 1 Values

Response Type	DC Gain (dB)	Time Constant (s)	Pole Location (rad/s)
Sinusoidal	6.1798	0.07958	12.5650
Step	6.7407	0.0442	22.6116

TABLE II
Researcher 2 Values

Response Type	DC Gain (dB)	Time Constant (s)	Pole Location (rad/s)
Sinusoidal	6.2140	0.07957	12.5664
Step	6.3904	0.0424	23.5706

B. Theoretical Responses

Based on the parameters extracted from the experimental bode plots, the theoretical transfer functions were generated employing MATLAB's modeling tools. As both Researcher 1 and Researcher 2 values for the DC gain and time constant are similar, the transfer function models output almost coincident bode plots, as shown in Fig. 5. Both data sets show a change in slope, from 0 to -20 decibels per decade for the magnitude plot and from 0 to -45 degrees per decade for the phase plot, at the stipulated pole locations stated in Table I and Table II.

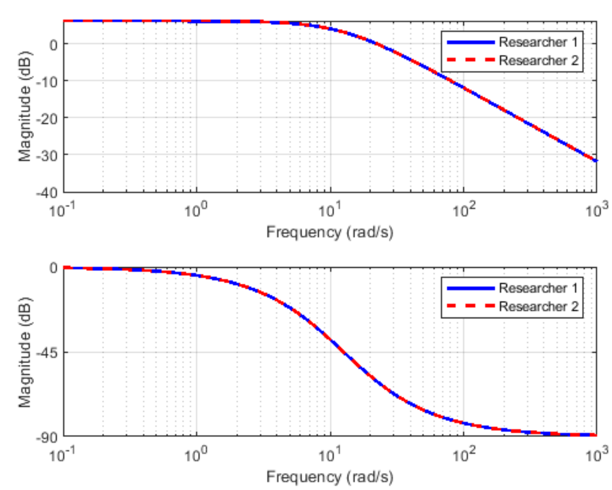

Fig. 5. Theoretical magnitude and phase bode plot of the sinusoidal response with a logarithmic frequency scale created using MATLAB.

Furthermore, theoretical transfer function models were also generated to simulate the step response of the system based on parameters obtained from the experimental step response of the system. The theoretical transfer functions model the step responses, starting at a velocity of 0 radians per second and increasing to 6.38 and 6.73 radians per second respectively for Researcher 1 and Researcher 2, as shown in Fig. 6.

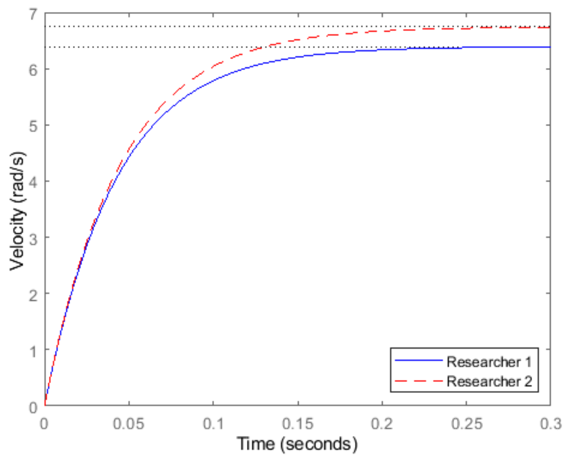

Fig. 6. Theoretical step response plot in the time domain created using MATLAB.

IV. Discussion

The discussion section of this report is to explore the two methods used to find the transfer function of a predesigned dynamic system. With the frequency-domain approach that sent sinusoidal wave input signals, experimental data was used to create bode plots for both the DC gain and phase shift of the system. The time-domain approach sent square wave input signals to create a step response graph where both the gain and time constant can be extracted. These methods provide enough information to generate an experimental and theoretical transfer function in which they will be compared for the two sets of data gathered by the researchers.

A. *Theoretical Vs. Experimental Analysis*

Upon the generation of the theoretical graphs, it can be seen for the frequency response method that they follow the trend of the experimental graphs for both the researchers and their respective data. In both cases, the data holds a steady magnitude of about 6.2 decibels until they reach the pole in which it begins to decline at the different slopes. It is at this pole where the theoretical data deviates from the experimental data. For both researchers, the experimental data saw a downwards slope of -10 decibels per decade once it reaches the pole at 12.565 radians per second and for the theoretical data there was a slope of -20 decibels per decade beginning at the location of the pole, 12.566 radians per second. This observation occurs due to the imperfections and environmental factors (e.g., friction) existing in the motor flywheel system which interferes with the data acquisition.

The theoretical graph for the step response method shows the same trend as the experimental data, however it begins at an initial position of zero radians per second. The experimental data which represents the real response to the input command signals requires some time for the motor switch directions. This discrepancy is not seen with the sinusoidal input signals as there is some time to accelerate and decelerate when switching directions.

B. *Researcher Data Comparison*

In the analysis of the bode plot and step response data provided by both researchers, certain differences are evident when comparing experimental findings between Researcher 1 and Researcher 2. For the bode plot, the DC gain is affected by the chosen motor effort by each researcher. As Researcher 1 used 30% motor effort, the DC gain was slightly lower than the DC gain from Researcher 2, who used 40% motor effort. Moreover, Researcher 2’s experimental bode plot displayed a stronger attenuation in magnitude and longer delay in phase at higher frequencies. Likewise, for the step response, Researcher 2’s findings depict a faster rise time than Researcher 1’s findings. This is due to Researcher 2’s step response having a smaller time constant, which is inversely proportional to the rise time.

C. *Limitation*

This report utilized a motor flywheel system in which many natural limitations in the performance of the hardware brought in the biggest discrepancies and inaccuracies. The existing friction within the system interfered with the acquisition of data creating excess noise and disturbances. This was especially observed to occur in higher frequencies as it is assumed that the hardware had difficulties keeping up with the command input signals. There was also trouble with the transfer of large sets of data which occurred due to the lack of a high-performance computer.

D. *Recommendations*

As with any experiment, improvements can be made to provide more accurate results such as decreasing the amount of mechanical friction between the moving parts within the system. It would also benefit the researchers if more time was spent accumulating the sinusoidal response to receive richer or more accurate data. The hardware can also be updated to include sharper sending of signals as well as stiffer material to nullify the effects of any external or internal forces such as wind or vibration.

References

[1] S. Ridgeway, EML 4314C - Spring 2024 - Lab 1 Assignment, Spring 2024.

[2] Pendulum v2.1 Assembly. Out of the Box Electronics and Robotics LLC, 2024.

[3] S. Ridgeway, “EML 4314C Spring 2023 - Lecture 3 slides,” Spring. 2024

[4] S. Ridgeway, “EML 4314C Spring 2023 - Lecture 4 slides,” Spring. 2024