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1929 Stadium Rd., Room 0312
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Dear Mr. Rinzel,

This lab focuses on determining the applicability of a double pipe heat exchanger. Evaluating the performance of the apparatus is completed through the calculation of the Log Mean Temperature Difference (LMTD); next, analysis of the overall heat transfer coefficient (UA) includes finding the heat transfer rate, the convective heat transfer coefficient, internal flow properties. Finally, the experimental and theoretical data can be compared to obtain the fouling factor and compute its associated uncertainty for different configurations.

Heat exchangers are devices designed to enable the process of heat transfer between two or more fluids, and certain configurations are utilized depending on flow arrangement and construction. Two flow configurations were used in this lab: parallel flow and counterflow. Each configuration makes use of a fluid at varying hot and cold temperatures. The piping utilized in the heat exchanger features a concentric tubing setup, allowing both convective and conductive heat transfer modes. It is assumed that there were no heat losses due to the outer pipe being exposed to atmospheric conditions. The inlet and outlet temperatures of both flows were used to represent an effective temperature between the fluids, e.g., LMTD. With flow properties, the heat transfer rate divided by the LMTD provides the experimental UA. To predict UA, internal convective heat transfer effects need to be considered. Assuming the system is working at steady state, the Reynolds number, the Prandtl number, and the friction factor were calculated to find the Nusselt number, which is proportional to the convective heat transfer coefficient.

A set of six thermistors on the heat exchanger were used to measure the temperatures at inlet, middle, and outlet of the concentric pipe system. Additionally, two flowmeters for both the inner and outer pipe were used to measure the flowrate of each fluid; the flowmeters were calibrated before collecting data to correctly relate the voltage to LPM. To acquire and visualize the data, a LabVIEW VI processed the readings based on the voltage fluctuations. By measuring the amount of the fluid sensed by the flowmeter and discharged in a graduated cylinder in a set time frame, calibration constants were derived from the linear relation between the voltage and LPM. Two data sets were recorded for both parallel and counter flow configurations, one where the hot and cold fluids had an equal flow of 3 LPM, and the other where hot fluid flows at 3 LPM and cold fluid flows at 6 LPM. While conducting the lab, safety glasses, hearing protection and closed toed shoes were enforced to ensure safety.

The objective of this laboratory was to assess the performance of the double pipe heat exchanger in different configurations and flow conditions. For the parallel flow configuration, the LMTD was 16.09 K and 16.44 K for equal flow and unequal flow conditions, respectively. The experimental UA value fell between 112-137 W/K; conversely, the theoretical UA fell between 89-151 W/K. For the counter-flow configuration, the LMTD was 16.88 K and 16.97 K for equal flow and unequal flow conditions, respectively. Furthermore, the experimental UA value fell between 110-155 W/K, and the theoretical UA value fell between 90-156 W/K. The fouling factor, solely calculated for the unequal counter flow configuration, was found to be 3.15×10^{-6} $K \cdot m^2/W$. Between the values for each layout, the counter flow configuration generally yielded larger values than the parallel flow configuration. Values for UA roughly fell in the category of basic applications of heat exchangers.

Sincerely,

Conor Bowman   Alex Carr   Allison Porras   David Reyes-Tobar

Table 1: Calibration constants for the flowmeter were experimentally determined for both hot and cold flow. These values were calculated by recording voltage and volumetric flow rate over the range of the valve's opening. At each randomly chosen valve open position, the voltage was recorded along with the volume of water pumped from the system over a 30 second window. After obtaining values across the valve open position, the points were plotted to obtain a calibration curve.

Experimental Unit Location	**NSC 0312**		
Flowmeter calibration constants	**A (LPM/V^2)**	**B (LPM/V)**	**C (LPM)**
Cold flow	0	1.7709	-0.1166
Hot flow	0	1.8656	-0.0042

Table 2: Raw experimental data obtained from LabView VI for each of the 4 experiments. Two runs for Counter Flow using different Hot/Cold flow rates and the same for Parallel Flow. As shown in the table, the unequal flow rates had a significant impact on heat exchanger capability, which is to be expected.

	COLD	**HOT**	**COLD**	**COLD**	**HOT**	**HOT**
	Flow rate (LPM)	**Flow rate (LPM)**	**T_{in} (°C)**	**T_{out} (°C)**	**T_{in} (°C)**	**T_{out} (°C)**
Counter flow	3.04	3.08	28.3	36.6	54.0	44.6
Counter flow	6.02	3.03	27.9	34.1	54.5	41.8
Parallel flow	3.05	3.03	27.9	35.9	54.1	44.9
Parallel flow	5.80	3.00	28.1	33.2	54.5	42.6

Table 3: Experimental values for the LMTD and UA for the two flow conditions for each heat exchanger configuration. Theoretical values for UA and the fouling factor are also listed for the two flow conditions for each heat exchanger configuration.

	COLD	**HOT**				
	Flow rate (LPM)	**Flow rate (LPM)**	**LMTD (K)**	**UA experimental (W/K)**	**UA theoretical (W/K)**	**R_f" (K·m^2/W)**
Counter flow	3.04	3.08	16.88	110.9	90.0	N/A
Counter flow	6.02	3.03	16.97	155.2	156.5	3.15×10^{-6}
Parallel flow	3.05	3.03	16.09	112.3	89.0	N/A
Parallel flow	5.80	3.00	16.44	137.0	151.7	N/A

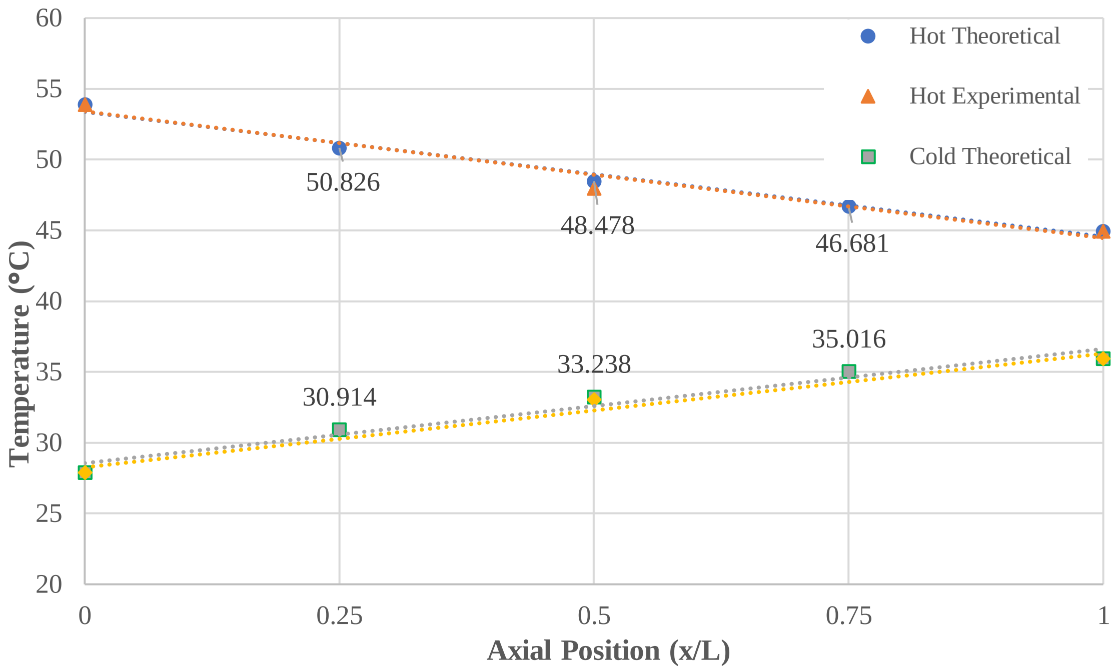

Figure 1: The experimental and theoretical hot and cold temperatures versus axial position in a parallel equal flow rate configuration. The experimental temperatures were recorded during the data collection process at positions of 0, 0.5, and 1. The theoretical temperatures were calculated using the ε-NTU method at axial positions of 0.25, 0.5, and 0.75. The hot water linearly decreased in temperature while the cold water increased in temperature in the parallel flow configuration. The maximum relative error between experimental and theoretical temperatures is 1.09%.

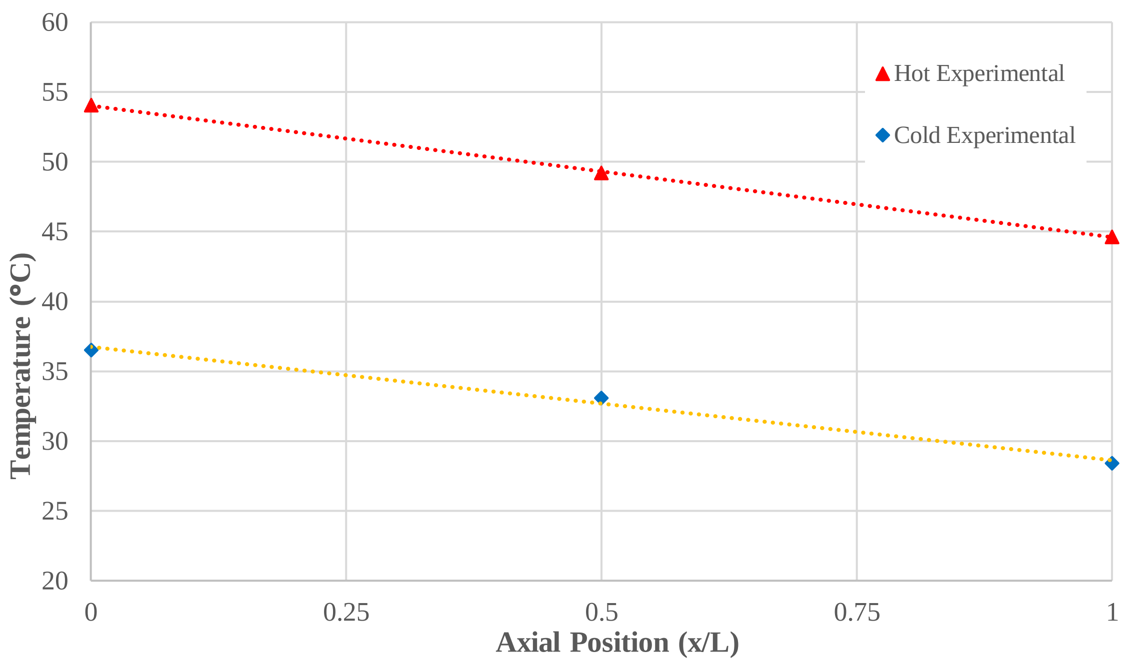

Figure 2: The experimental hot and cold temperatures versus axial position in a counter equal flow rate configuration. The experimental temperatures were recorded during the data collection process at axial positions of 0, 0.5, and 1. The hot and cold water linearly decreased in temperature in the counter flow configuration. The hot water decreases at a rate 1 ° C greater than the cold water.

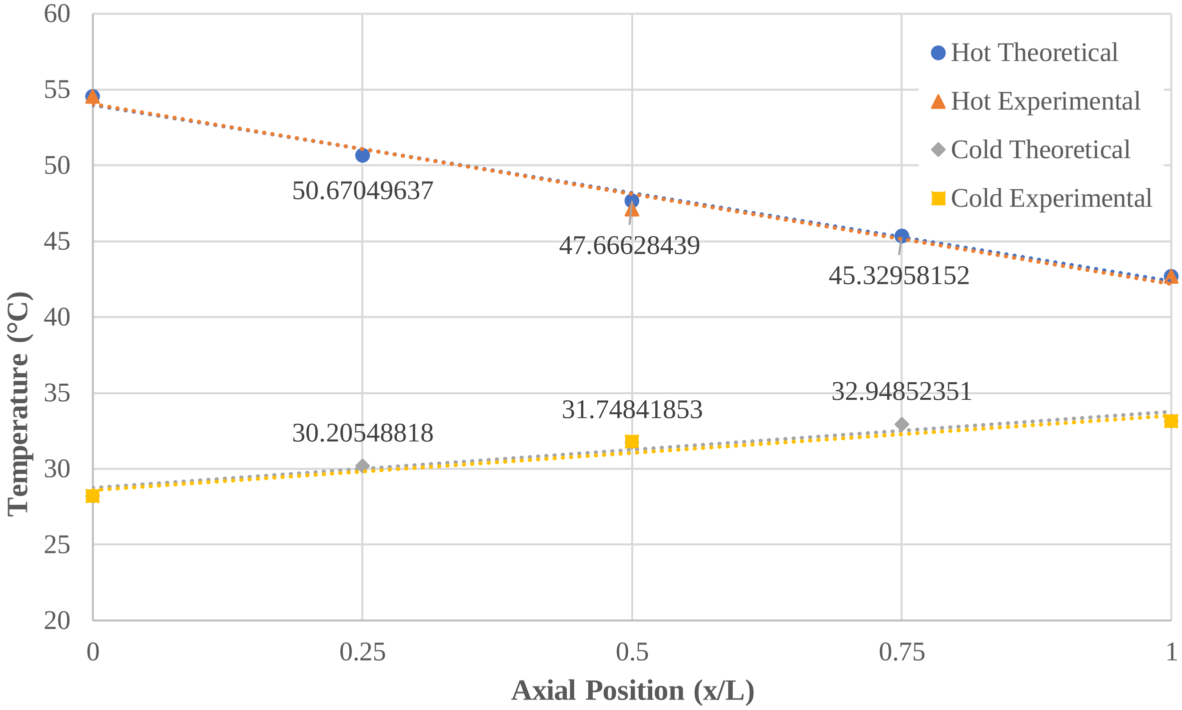

Figure 3: Experimental and theoretical values are presented for parallel flow in a concentric tube heat exchanger; the unequal flow rates for this data were 3 liters/minute for the hot fluid and 6 liters/minute for the cold fluid. Using the ε-NTU method, intermediate temperatures were calculated for the theoretical data to correspond with x/L = ¼, ½, and ¾. The graph models an appropriate configuration for parallel flow.

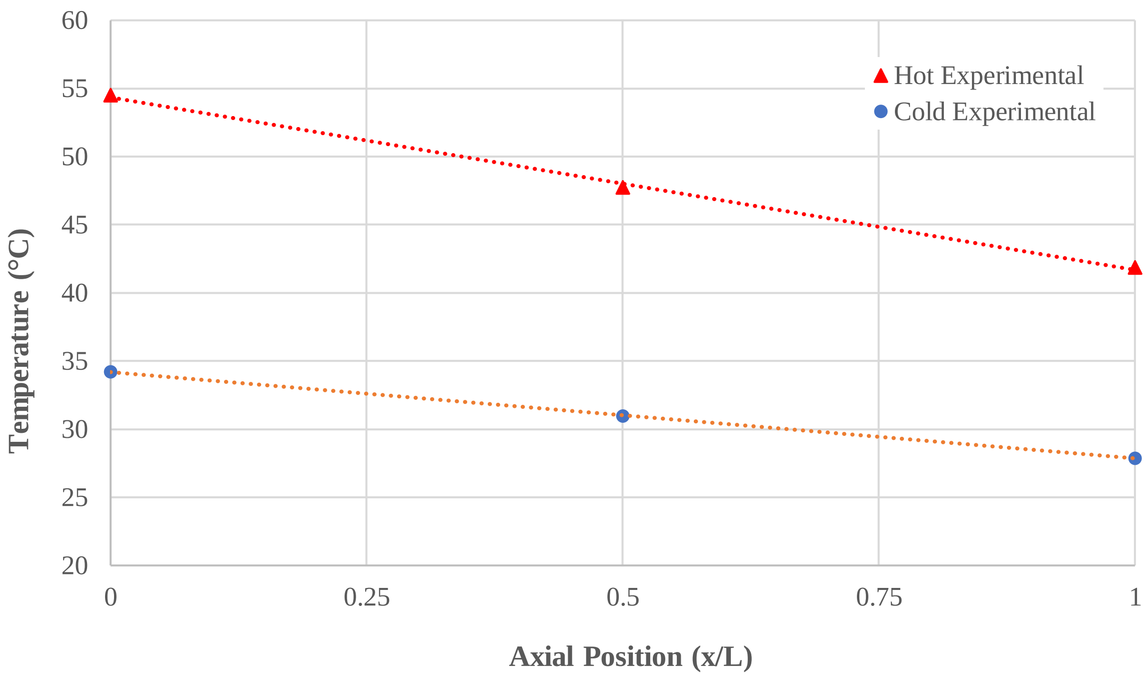

Figure 4: Experimental values are presented for counter flow in a concentric tube heat exchanger; the unequal flow rates for this data were 3 liters/minute for the hot fluid and 6 liters/minute for the cold fluid. The graph models an appropriate configuration for counter flow.

Table 4: Table includes values of uncertainties associated with UA for all four experimental runs. The following uncertainties utilize data points from both hot and cold pipes. In this table, the higher the uncertainty of the subcomponents the more of an impact it has on the overall UA uncertainty. As shown in the table, the uncertainties in flow rate of hot and cold pipes are by far the biggest contributor to UA uncertainty. The least important contributors to UA uncertainty are the C_p values. The uncertainty in UA is determined via the Root Sum Square (RSS) method using the subcomponents listed in table, which has an example calculation later in report.

Condition	U_{UA}	$U\dot{m}_h$	UC_{ph}	$UT_{h,i}$	$UT_{h,o}$	$U\dot{m}_c$	$UC_{p,c}$	$UT_{c,o}$	$UT_{c,i}$
Equal Counter Flow	3.51	0.77	0.0003	0.55	0.57	3.23	0.0006	0.43	0.68
Unequal Counter Flow	4.61	3.39	0.0011	0.46	0.49	2.96	0.0005	0.47	0.63
Equal Parallel Flow	3.75	2.59	0.0034	0.50	0.54	2.48	0.0005	0.48	0.65
Unequal Parallel Flow	4.35	2.62	0.0032	0.52	0.56	3.30	0.0005	0.43	0.62

Sample Calculations

The Log Mean Temperature Difference (LMTD) is the appropriate average temperature difference between inlet and outlet temperatures of both the hot and cold fluids. Changes in temperature, known as ΔT_1 and ΔT_2, are different depending on which configuration the heat exchanger is on, parallel or counter flow. For parallel flow, ΔT_1 and ΔT_2 are defined as,

$$\Delta T_1 = T_{h,i} - T_{c,i}$$
$$\Delta T_2 = T_{h,o} - T_{c,o}$$

Course Notes

where the subscripts *h* and *c* refer to the hot and cold temperatures and the subscripts *i* and *o* refer to the inlet and outlet temperatures of both the cold and hot flows. For counter flow, ΔT_1 and ΔT_2 are defined as,

$$\Delta T_1 = T_{h,i} - T_{c,o}$$
$$\Delta T_2 = T_{h,o} - T_{c,i}$$

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With values know for ΔT_1 and ΔT_2, LMTD, also known as ΔT_{lm}, can be defined as,

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\frac{\Delta T_1}{\Delta T_2}\right)} = \frac{\Delta T_2 - \Delta T_1}{\ln\left(\frac{\Delta T_2}{\Delta T_1}\right)}$$

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Take the case where the heat exchanger is in a counter flow configuration where the cold fluid is flowing at around 6 LPM and the hot fluid is flowing at around 3 LPM. Therefore, ΔT_1, ΔT_2, and ΔT_{lm} have the following values.

$$\Delta T_1 = (273.15\ K + 54.54\ °C) - (273.15\ K + 34.13\ °C) = 20.41\ K$$

$$\Delta T_2 = (273.15\ K + 41.82\ °C) - (273.15\ K + 27.87\ °C) = 13.95\ K$$

$$\Delta T_{lm} = \frac{20.41\ K - 13.95\ K}{\ln\left(\frac{20.41\ K}{13.95\ K}\right)} = 16.97\ K$$

The overall heat transfer coefficient, U, multiplied by both the surface area of the pipe, A, and the LMTD produces the heat transfer rate, q. Therefore, the experimental UA can defined as,

$$UA = U_o A_o = U_i A_i = \frac{q}{\Delta T_{lm}}$$

Course Notes

Now, it is necessary to calculate the heat transfer rate. The heat transfer rate can be calculated separately for both the hot and cold flows using the mass flow rate $\dot{m}$, the specific heat capacity c_p, the inlet and outlet temperatures for each flow.

$$q_h = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o})$$
$$q_c = \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i})$$

Course Notes

As there is some error included in each value used in the previous equations, q_h and q_c are not exactly the same value, thus, the average of both is used.

$$q = \frac{q_h + q_c}{2}$$

The volumetric flow rate needs to be converted to mass flow rate using density. To find the values of ρ and c_p for both the hot and cold flows, the average between the inlet and outlet temperatures is used to interpolate between values provided by Table A.6 in the *Fundamentals of Heat and Mass Transfer, 8^th^ edition* textbook.

$$q_h = 3.03\frac{liters}{min}\times\frac{m^3}{1000\ liters}\times\frac{min}{60\ s}\times\ 988.6\frac{kg}{m^3}\times 4180.5\frac{J}{kg\cdot K}\times(54.54-41.82)\ K = 2659.6\ W$$

$$q_c = 6.02\frac{liters}{min}\times\frac{m^3}{1000\ liters}\times\frac{min}{60\ s}\times\ 995.4\frac{kg}{m^3}\times 4178\frac{J}{kg\cdot K}\times(34.13-27.87)\ K = 2610.6\ W$$

$$q = \frac{2659.6\ W + 2610.6}{2} = 2635.1\ W$$

$$UA = \frac{2635.1\ W}{16.97\ K} = 155.2\frac{W}{K}$$

For the calculation of theoretical overall heat transfer coefficient, conduction and convection resistances between fluids separated by cylindrical walls are taken into consideration. For concentric tubular heat exchangers, the theoretical overall heat transfer coefficient is defined as

$$\frac{1}{UA} = \frac{1}{U_oA_o} = \frac{1}{U_iA_i} = \frac{1}{h_iA_i} + \frac{R_{f,i}''}{A_i} + \frac{\ln\left(D_o/D_i\right)}{2\pi kL} + \frac{R_{f,o}''}{A_o} + \frac{1}{h_oA_o}$$

Course Notes

Assuming that the fouling factor R_f'' is zero, the equation becomes,

$$\frac{1}{UA} = \frac{1}{h_iA_i} + \frac{\ln\left(D_o/D_i\right)}{2\pi kL} + \frac{1}{h_oA_o}$$

The area A_i is the surface areas of inner pipe using the inner diameter D_i and the area A_o is the surface area of the inner pipe using the outer diameter D_o. The length L is the length of the pipe. The thermal conductivity k is the thermal conductivity of the material the inner pipe is made out of, which is copper. The convective heat transfer coefficient, h, depends on the flow conditions of the hot and cold fluids. First, it is necessary to calculate the Reynolds number and Prandtl number of each flow to determine if the flow satisfies certain conditions to use some equations that calculate the Nusselt number, which is related to the convective heat transfer coefficient. The Reynolds number is defined as

$$Re = \frac{\rho u_m D_h}{\mu}$$

Eq. 8.1, Fundamentals of Heat and Mass Transfer 8th Edition

where μ is the dynamic viscosity of the fluid, u_m is the mean fluid velocity over the pipe cross section defined as

$$u_m = \frac{\dot{\forall}}{A_c}$$

Course Notes

and D_h is the hydraulic diameter defined as

$$D_h = \frac{4A_c}{P}$$

Eq. 8.66, Fundamentals of Heat and Mass Transfer 8th Edition

where A_c and P are the flow cross-sectional area and the wetted perimeter, respectively. The hydraulic diameter is used to apply the results to an effective diameter. For the concentric tube configuration, the inner pipe has a hydraulic diameter equal to

$$D_{h,i} = \frac{4\left(\frac{\pi D_i^2}{4}\right)}{\pi D_i} = D_i$$

In the other hand, the outer pipe has a hydraulic diameter equal to

$$D_{h,o} = \frac{4\left(\frac{\pi}{4}\right)\left(D_o^2 - D_i^2\right)}{\pi D_o + \pi D_i} = D_o - D_i$$

Calculating the Reynolds number for the hot and cold flows in the unequal counter flow configuration, respectively,

$$u_{m,i} = \frac{\left(3.03\,\frac{\cancel{liters}}{\cancel{min}} \times \frac{m^{\cancel{3}}}{1000\,\cancel{liters}} \times \frac{\cancel{min}}{60\,s}\right)}{\frac{\pi(0.0109^2)}{4}\,\cancel{m^2}} = 0.542\,\frac{m}{s}$$

$$Re_i = \frac{988.6\frac{\cancel{kg}}{\cancel{m^3}} \times 0.542\frac{\cancel{m}}{\cancel{s}} \times (0.0109\,\cancel{m})}{0.000564\,\frac{\cancel{N \cdot s}}{\cancel{m^2}}} = 10355.7$$

$$u_{m,o} = \frac{\left(6.02\,\frac{\cancel{liters}}{\cancel{min}} \times \frac{m^{\cancel{3}}}{1000\,\cancel{liters}} \times \frac{\cancel{min}}{60\,s}\right)}{\frac{\pi(0.0225^2 - 0.0128^2)}{4}\,\cancel{m^2}} = 0.373\,\frac{m}{s}$$

$$Re_o = \frac{995.4\frac{\cancel{kg}}{\cancel{m^3}} \times 0.373\frac{\cancel{m}}{\cancel{s}} \times (0.0225\,\cancel{m} - 0.0128\,\cancel{m})}{0.000784\frac{\cancel{N \cdot s}}{\cancel{m^2}}} = 4598.1$$

Moreover, the Prandtl number is defined as

$$Pr = \frac{C_p \mu}{k}$$

where k is the thermal conductivity of the fluid. Calculating the Prandtl number for the hot and cold flows in the unequal counter flow configuration, respectively,

$$Pr_i = \frac{4180.5\frac{\cancel{J}}{\cancel{kg \cdot K}} \times 0.000564\frac{\cancel{N \cdot s}}{\cancel{m^2}}}{0.641\frac{\cancel{W}}{\cancel{m \cdot K}}} = 3.676$$

$$Pr_o = \frac{4178\frac{\cancel{J}}{\cancel{kg \cdot K}} \times 0.000784\frac{\cancel{N \cdot s}}{\cancel{m^2}}}{0.619\frac{\cancel{W}}{\cancel{m \cdot K}}} = 5.291$$

To find the values of μ and k for both the hot and cold flows, the average between the inlet and outlet temperatures is used to interpolate between values provided by Table A.6 in the *Fundamentals of Heat and Mass Transfer, 8th edition* textbook.

As the Reynolds number and Prandtl number previously calculated are between $3000 \leq Re \leq 5 \times 10^6$ and $0.5 \leq Pr \leq 2000$, the following Nusselt number can be used,

$$Nu = \frac{(f/8)(Re - 1000)\, Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$$

Eq. 8.62, Fundamentals of Heat and Mass Transfer 8th Edition

where f is the friction factor defined as

$$f = (0.790 \ln Re - 1.64)^{-2}$$

Eq. 8.21, Fundamentals of Heat and Mass Transfer 8th Edition

With the Nusselt number found, the convective heat transfer coefficient can be found to be

$$h = \frac{Nu \cdot k}{D_h}$$

Eq. 8.69, Fundamentals of Heat and Mass Transfer 8th Edition

For the unequal counter flow configuration,

$$f_i = (0.790 \ln(10355.7) - 1.64)^{-2} = 0.0312$$

$$Nu_i = \frac{(0.0312/8)(10355.7 - 1000)\,(3.676)}{1 + 12.7(0.0312/8)^{1/2}(3.676^{2/3} - 1)} = 63.95$$

$$h_i = \frac{63.95 \times 0.641 \frac{W}{m \cdot K}}{0.0109\, m} = 3762.9 \frac{W}{m^2 \cdot K}$$

$$f_o = (0.790 \ln(4598.1) - 1.64)^{-2} = 0.0396$$

$$Nu_o = \frac{(0.0396/8)(4598.1 - 1000)\,(5.291)}{1 + 12.7(0.0396/8)^{1/2}(5.291^{2/3} - 1)} = 33.45$$

$$h_o = \frac{33.45 \times 0.619 \frac{W}{m \cdot K}}{(0.0225\, m - 0.0128\, m)} = 2133.8 \frac{W}{m^2 \cdot K}$$

Now that the convective heat transfer coefficients have been determined, the theoretical UA is as it follows

$$\frac{1}{UA} = \frac{1}{3762.9 \frac{W}{\cancel{m^2} \cdot K} \times (\pi \times 0.0109\, \cancel{m} \times 3.05\, \cancel{m})} + \frac{\ln\left(0.0128\, \cancel{m} / 0.0109\, \cancel{m}\right)}{2\pi \times 401 \frac{W}{\cancel{m} \cdot K} \times 3.05\, \cancel{m}} + \frac{1}{2133.8 \frac{W}{\cancel{m^2} \cdot K} \times (\pi \times 0.0128\, \cancel{m} \times 3.05\, \cancel{m})}$$

$$UA = 156.5 \frac{W}{K}$$

Previously set equal to zero in the calculations of the theoretical UA, its value may not be ignored as its effects are included in the calculations of the experimental UA. Using the relation between both the experimental UA and theoretical UA, the following relationship can be used

$$\frac{1}{(UA)_{actual}} = \frac{1}{(UA)_{theoretical}} + \frac{R''_{f,i}}{A_i} + \frac{R''_{f,o}}{A_o}$$

Course Notes

Assuming $R''_{f,i} = R''_{f,o} = R''_f$ and solving for the fouling factor, it is found that

$$R''_f = \frac{\frac{1}{(UA)_{actual}} - \frac{1}{(UA)_{theoretical}}}{\left(\frac{1}{A_i} - \frac{1}{A_o}\right)}$$

$$R''_f = \frac{\frac{1}{155.2\frac{W}{K}} - \frac{1}{156.5\frac{W}{K}}}{\frac{1}{\pi \times 0.0109\ m \times 3.05\ m} + \frac{1}{\pi \times 0.0128\ m \times 3.05\ m}} = 3.15 \times 10^{-6} \frac{K \cdot m^2}{W}$$

The temperature of the pipe at any axial position (x/L) can be found by using the ε-NTU method for a heat exchanger. The outlet temperature, $T_{h,o}$, is calculated using the following equation where $T_{h,i}$ is the inlet temperature, ε is the effectiveness of the heat exchanger, q_{max} is the maximum heat transfer rate, $\dot{m}$ is the mass flow rate, and C_p is the specific heat capacity of the water. $T_{h,o} = T_{h,i} - \frac{\varepsilon q_{max}}{\dot{m} C_p}$

$$T_{h,o} = T_{h,i} - \frac{\varepsilon q_{max}}{\dot{m} C_p}$$ *Course Notes*

The maximum heat transfer rate, q_{max}, can be calculated using the following equation where ΔT_1 is the temperature change between the hot inlet temperature, $T_{h,i}$, and the cold inlet temperature, $T_{c,i}$ and C_{min} is the minimum heat capacity of the water.

$$q_{max} = \Delta T_1 (C_{min})$$ *Eqn. 11.18, Fundamentals of Heat and Mass Transfer 8^{th} Edition*

The minimum heat capacity, C_{min}, can be found by multiplying the mass flow rate, $\dot{m}$, and the specific heat capacity, C_p. That equation is represented below.

$$C_{min} = \dot{m} C_p$$ *Course Notes*

The number of transfer units, NTU, is a dimensionless parameter that can be calculated using the following equation where UA is the theoretical overall heat transfer coefficient, $\frac{x}{L}$ is the axial position, and C_{min} is the minimum heat capacity.

$$NTU = \frac{UA(\frac{x}{L})}{C_{min}}$$ *Eqn. 11.24, Fundamentals of Heat and Mass Transfer 8^{th} Edition*

The heat capacity ratio, C_r, is needed to find the effectiveness of the heat exchanger and is the ratio of the minimum heat capacity, C_{min}, and the maximum heat capacity, C_{max}.

$$C_r = \frac{C_{min}}{C_{max}}$$ *Course Notes*

Finally, the effectiveness of the heat exchanger for parallel flow can be calculated using the following equation where NTU is the number of transfer units and C_r is the heat capacity ratio.

$$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{(1 + C_r)}$$ *Eqn. 11.29a, Fundamentals of Heat and Mass Transfer 8^{th} Edition*

The following is an example of how to calculate the theoretical temperature of the hot water for a parallel flow at an axial position of $\frac{1}{2}$.

$$q_{max} = \Delta T_1(C_{min}) = 26.016\ \cancel{°C} * 208.932 \frac{\cancel{kg * J}}{\cancel{kg * K}} * (\frac{\cancel{K}}{\cancel{°C}} * \frac{W * \cancel{s}}{\cancel{J}}) = 5435.574\ W$$

$$C_{min} = \dot{m}C_p = 0.0499 \frac{\cancel{kg}}{\cancel{s}} * 4179 \frac{\cancel{J}}{K * \cancel{kg}} * (\frac{W * \cancel{s}}{J}) = 208.932 \frac{W}{K}$$

$$NTU = \frac{UA(\frac{x}{L})}{C_{min}} = \frac{112.306 \frac{\cancel{W}}{\cancel{K}} * (\frac{1}{2})}{211.103 \frac{\cancel{kg * J}}{\cancel{s * kg * K}} * (\frac{\cancel{W * s}}{\cancel{J}})} = 0.265$$

$$C_r = \frac{C_{min}}{C_{max}} = \frac{208.932 \frac{\cancel{W}}{\cancel{K}}}{211.103 \frac{\cancel{W}}{\cancel{K}}} = 0.989$$

$$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{(1 + C_r)} = \frac{1 - \exp[-0.265 * (1 + 0.989)}{(1 + 0.989)} = 0.208$$

$$T_{h,o} = T_{h,i} - \frac{\varepsilon q_{max}}{\dot{m}C_p} = 53.894 - \frac{0.208 * 5435.574\ \cancel{W}}{208.932 \frac{\cancel{W}}{\cancel{K}} * (\frac{°C}{\cancel{K}})} = 48.478\ °C$$

Uncertainty for UA

Solving for uncertainty in UA, the equation for UA must be converted into terms used in the experiment for measured values.

$$UA = \frac{q}{\Delta T_{lm}} = \frac{\frac{[\dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) + \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i})]}{2}}{\frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}}$$

Converted into more experimental terms, this equation for UA is acquired for counter flow, unequal flow rates heat exchanger set up.

$$UA = \frac{\frac{[\dot{m}_h c_{p,h}(T_{h,i} - T_{h,o}) + \dot{m}_c c_{p,c}(T_{c,o} - T_{c,i})]}{2}}{\frac{(T_{hi} - T_{co}) - (T_{ho} - T_{ci})}{\ln\left({T_{hi} - T_{co}}/{T_{ho} - T_{ci}}\right)}}$$

Uncertainty in UA is calculated using the Root Sum Square Method. The equation for the uncertainty in UA is found below using partial derivatives of the equation above (example: $\frac{\partial UA}{\partial \dot{m}_h}$) along with measurement uncertainties (example: $U_{\dot{m}_h}$).

$$U_{UA} = \sqrt{\begin{array}{c}\left(\frac{\partial UA}{\partial \dot{m}_h} U_{\dot{m}_h}\right)^2 + \left(\frac{\partial UA}{\partial c_{p,h}} U_{c_{p,h}}\right)^2 + \left(\frac{\partial UA}{\partial T_{h,i}} U_{T_{h,i}}\right)^2 + \left(\frac{\partial UA}{\partial T_{h,o}} U_{T_{h,o}}\right)^2 + \\ \left(\frac{\partial UA}{\partial \dot{m}_c} U_{\dot{m}_c}\right)^2 + \left(\frac{\partial UA}{\partial c_{p,c}} U_{c_{p,c}}\right)^2 + \left(\frac{\partial UA}{\partial T_{c,o}} U_{T_{c,o}}\right)^2 + \left(\frac{\partial UA}{\partial T_{ci}} U_{T_{c,i}}\right)^2\end{array}}$$

The following is an example of uncertainty in UA is from counter-flow, uneven flow rates. Starting the equation, below is an example of determining the component $\frac{\partial UA}{\partial \dot{m}_h}$.

$$\frac{\partial UA}{\partial \dot{m}_h} = \frac{-C_{p,h} * \ln(\frac{T_{hi} - T_{co}}{T_{ho} - T_{ci}})(T_{hi} - T_{co})}{2 * (T_{hi} - T_{co}) - (T_{ho} - T_{ci})}$$

$$\frac{\partial UA}{\partial \dot{m}_h} = \frac{-4178 \frac{J}{kg * K} * \ln(\frac{20.4\ \cancel{K}}{13.9\ \cancel{K}})(327.7\cancel{K} - 307.3\cancel{K})}{2 * (327.7\cancel{K} - 307.3\ \cancel{K}) - (315.0\ \cancel{K} - 301.0\ \cancel{K})} = -120.9 \frac{J}{kg * K}$$

The next component is the measurement uncertainty of $\dot{m}_h$. This is determined based off manufacturer specifications of flowrate.

$$U_{\dot{m}_h} = 0.028 \frac{kg}{s}$$

This example is incorporated across all components to compile into equation for uncertainty in UA.

$$U_{UA} = \sqrt{\begin{array}{c}\left(-120.9\ \frac{J}{kg * K} * 0.028 \frac{kg}{s}\right)^2 + \left(-0.011 \frac{J}{kg * K} * 0.1\ K\right)^2 + \left(4.628 \frac{J}{kg * K} * 0.1\ K\right)^2 + \\ \left(-4.865 \frac{J}{kg * K} * 0.1\ K\right)^2 + \left(-105.671 \frac{J}{kg * K} * 0.028 \frac{ks}{s}\right)^2 + \left(0.005 \frac{J}{kg * K} * 0.1 \frac{J}{kg * K}\right)^2 + \\ \left(4.673 \frac{J}{kg * K} * 0.1\ K\right)^2 + \left(-6.264 \frac{J}{kg * K} * 0.1\ K\right)^2\end{array}}$$

$$U_{UA} = 4.613$$

Lab Report Participation Log

Name	Date	Hours Worked	Description of Tasks Performed
Alli Porras	6/20	1	Set up report document with sections + descriptions
Everyone	6/22	1	Everyone met up to divide tasks
Conor Bowman	6/28	3	Finished data table and started Uncertainty in UA
David Reyes	7/2	3	Calculate LMTD, UA experimental, UA theoretical, interpolate to find fluid properties, and set up equations in Excel
Conor Bowman	7/6	5	Finished uncertainty in UA table and sample equations.
Alli Porras	7/5	2	Added equations for sample calculations
Alli Porras	7/6	2	Worked on data for plot of temperature vs. axial position (unequal flow rates)
Alex Carr	7/6	1	Worked on plotting temperature vs axial position (equal flow rates)
Conor Bowman	7/7	1	Clean up captions.
Alli Porras	7/7	2	Finish graphs for unequal flow rates, add captions, and review rest of report for consistency
David Reyes	7/7	3	Finish calculations and tabulate them, start and finish sample calculations, and write letter
Alex Carr	7/7	3	Finished graphs for equal flow rates and completed theoretical temperature sample calc
	Total Hours:	27	